Scalar and vector product

This article contains the theoretical minimum in the field of vector algebra for the subject Biophysics and Medical Physics at the 1st Faculty of Medicine and includes the concepts contained in the exam questions for this subject.

Vector

In the general case, a vector is an ordered tuple of numbers (coordinates or vector components). In the Cartesian coordinate space, the three components of the vector correspond to the intersection of the vector in the respective coordinate axes. The magnitude (length) of a vector can be calculated as the square root of the sum of the squares of its coordinates.

$$\vec{v} = (v_x, v_y, v_z)$$

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Basic operations with vectors

The sum of vectors is a vector given by the coordinates that correspond to the sums of the corresponding coordinates. For graphic representation, the addition to the parallelogram is used, from which, using the cosine theorem, a relationship for calculating the magnitude of the sum of vectors follows.

$$\vec{w} = \vec{u} + \vec{v} = (u_x + v_x, u_y + v_y, u_z + v_z)$$

$w = \sqrt{u^2 + v^2 + 2uv\cos\varphi}$, where φ is the deviation of the vectors

The difference of vectors is analogously a vector given by the coordinates that correspond to the differences of the corresponding coordinates. In the graphic representation, we will use the so-called opposite vector, i.e. a vector of the same size but opposite direction (differs in sign). It is true that subtracting a vector is the same operation as adding the opposite vector.

$$\vec{w} = \vec{u} - \vec{v} = (u_x - v_x, u_y - v_y, u_z - v_z)$$

The product of a vector and a scalar is a vector whose direction is the same as the original (for a positive scalar) or opposite (for a negative scalar). By multiplying with a zero scalar, we get a zero vector. It can therefore be said that the absolute value of the scalar determines the change in size and its sign determines the change in direction of the resulting vector.

$$\vec{v} = k \cdot \vec{u} = (k \cdot u_x, k \cdot u_y, k \cdot u_z)$$

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Scalar and vector product

The scalar product of two vectors is the number that we get as a result of the sum of the products of the corresponding coordinates. The geometric meaning of the scalar product can be expressed as the magnitude of the projection of one vector onto another multiplied by the magnitude of the other.

$$k = \vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$

$$k = uv\cos\varphi$$

The vector product of two vectors is a vector perpendicular to the plane defined by both original vectors. Its direction relative to both vectors is determined by the right hand rule: If the direction from the palm to the bent fingers of the right hand is in the direction of the vector $\vec{u}$ the vector $\vec{v}$, then the raised thumb indicates the direction of the vector $\vec{w}$, which is their vector product. The geometric meaning of the magnitude of the vector product is the area of the parallelogram whose two adjacent sides correspond to the multiplied vectors.

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$$w = uv\sin\varphi$$

References

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- KLÍČ, Alois. *Matematika I ve strukturovaném studiu.* 2. edition. 2007. ISBN 978-80-7080-656-2.