Gradient

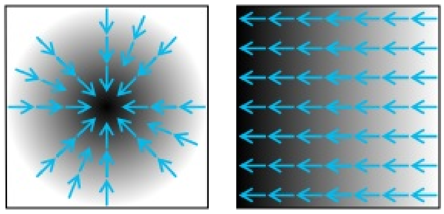
Gradient in general

A gradient in physics expresses the rate at which a physical quantity increases or decreases in proportion to changes in a given variable.

The gradient of a scalar function is a vector that at each point of the scalar field determines the direction of the fastest growth of the given function. The consequence of this is the possibility to describe the given scalar field as a gradient vector field.

Vector gradient array

The gradient of a scalar field is a vector field. At each point, the gradient is represented by a vector in the direction of which the given scalar function grows fastest, with the length of the vector representing the degree of steepness.



The figure shows the rate of climb (black area highest speed/white area lowest speed). The corresponding gradient is shown by blue arrows.

Converting a scalar field to a vector field

The scalar field tells us only the value of the quantity, not its direction (for example, measuring the temperature in a room). To obtain a vector, we use a gradient, which shows the direction in which the given vector changes in space and the largest increase in the given quantity. (An example can be a heat source in a room and a change in its intensity in space depending on the distance from the source.)

Example of conversion from scalar to vector field

Interpretation of the Gradient of the scalar field and conversion to a vector: khanovaskola (https://khanovaskola.cz/video/1128)

Operator Nabla

Nabla operator is a differential vector operator and is denoted by ∇. The name *nabla* originated from a Hebrew musical instrument with a triangular shape. An operator in mathematics is a prescription indicating an operation by which another function is assigned to a given function. Using the nabla operator also simplifies registration.

Mathematically, the nabla operator is defined as a vector of partial derivatives in the directions of individual coordinate axes:

$$\nabla := \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$$

So if the nabla operator is applied to the scalar function $u(x,y,z)$, we get the gradient:

$$\nabla u(x,y,z) = \left[\frac{\partial u(x,y,z)}{\partial x}, \frac{\partial u(x,y,z)}{\partial y}, \frac{\partial u(x,y,z)}{\partial z}\right] = \mathrm{grad}\, u(x,y,z)$$

Scalar functions are used in physics to describe scalar fields. The gradient is a vector that indicates the direction of greatest growth.

The Nabla operator can also be applied to vector functions, either in the sense of a scalar product (*divergence operator*, the result is a scalar function), or in the sense of a vector product (*rotation operator*, the result is again a vector function).

Examples of gradient

The gradient can be considered as a decisive factor according to which the particles will move and spread, and the force that will act on the given particles can be derived from its size.

Potential energy

Grad Ep determines the direction of the potential energy and is perpendicular to the equipotential surface. (The geometric locus of points with a given constant value of the quantity f, which is determined by the equation f(x,y,z) = const., is called an equipotential surface. Each point A of the field passes through exactly one equipotential surface.)[1]

Membrane potential

The electrical potential difference between the phospholipid bilayer membranes arises as a consequence of the voltage across the polarized membrane caused by the electrochemical gradient of the particles. The gradient causes the movement of ions across cell membranes and the subsequent distribution of charge across the membrane.

Electrochemical and concentration potential

Importance for cellular transport, especially for transport by membrane proteins, when the direction of transport is based on the prevailing vector of the gradient of electrochemical or concentration potential.

Other uses in potentials

Scalar Magnetic Potential

The scalar magnetic potential is used to describe the magnetic field, especially for permanent magnets. In a region of the same magnetization where there is no current,

$$\nabla \times \mathbf{H} = 0,$$

hence one can define the *magnetic scalar potential* ψ as

$$\mathbf{H} = -\nabla\psi.$$

Gravitational Potential

The gravitational potential gradient is defined as the rate of change of the gravitational potential with distance from the field of action. This is equal to the gravitational field strength at that point. The negative value of the gradient here determines the strength of the field.

Links

Related articles

- Potencial

External links

- gradient (mathematics)
- operator
- nabla
- vector
- magnetic pole
- magnet

Used sources

- Intenzita a potenciál, Katedra Fyziky FEL ČVUT (http://fyzika.feld.cvut.cz/~kriha/VirtLab/IntPot.pdf)
- Fyzika II. VŠCHT - Ústav techniky a měřící techniky (https://ufmt.vscht.cz/)
- The Gradient, HyperPhysics (http://hyperphysics.phy-astr.gsu.edu/hbase/gradi.html)
- Magnetic scalar potential, Wikipedia (https://en.wikipedia.org/wiki/Magnetic_scalar_potential)
- Gravitační potenciální gradient (http://www.schoolphysics.co.uk/age16-19/Mechanics/Gravitation/text/Gravitational_potential_gradient/index.html)

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BEDNAŘÍK, Michal. *Fyzika 1.* 1. edition. České vysoké učení technické, 2011. ISBN 978-80-01-04834-4.