Electrical impedance/NMgr

Electrical impedance is an extension of the term electrical resistance to situations where alternating current flows through an environment. The simplest view of impedance is that it is the resistance to alternating current. The unit of impedance is the Ohm Ω, usually denoted by the letter **Z**. If the impedance is connected to a voltage U and a current I flows through it , its value is given by Ohm's law:

$$Z = \frac{U}{I}$$

Impedance of electrical elements

The basic electrical elements are the **resistor** , **capacitor** and **inductor** (coil) . The basic property of a resistor is electrical resistance, the basic property of a capacitor is capacity, and the basic property of an inductor is induction. These are of course idealized models, to emphasize this fact these terms are used and not the technical names resistor, capacitor and coil.

When calculating impedances, the frequency f is not usually used , but the circular frequency ω determined by the relation:

$$\omega = 2\pi \cdot f$$

Resistor Impedance

The impedance of the resistor itself is called resistance, denoted R . The resistance value does not depend on the frequency.

Capacitor impedance

The impedance of the capacitor is called **capacitance** , it is usually denoted by X C . The capacitance is inversely proportional to the capacitance C of the capacitor and inversely proportional to the frequency f of the applied voltage:

$$X_C = \frac{1}{\omega C}$$

Inductor Impedance

The impedance of the inductor is called **inductance** , it is usually denoted by X L . Inductance is directly proportional to the inductance L of the inductor and directly proportional to the frequency f of the current flowing through the inductor:

$$X_L = \omega L$$

Impedance of series connection of resistor and capacitor

Impedances cannot be added quite easily, for the impedance Z *of* the series connection of resistor R and capacitor C :

$$Z = \sqrt{R^2 - \frac{1}{(\omega C)^2}}$$

Impedance of parallel connection of resistor and capacitor

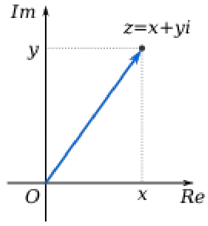
The relation for the impedance Z of a resistor and capacitor in parallel has a rather complex form, but it is worth noting because the parallel connection of a resistor and capacitor is a frequently used model of tissue impedance:

$$Z = \frac{\sqrt{R^2 + \omega^2 C^2 R^4}}{\omega^2 C^2 R^2 + 1}$$

Complex expression of impedance

The above relationships can be expressed very elegantly using complex numbers. The relationships known for connecting DC resistances then apply to connecting impedances. To distinguish the complex impedance from the absolute value of the impedance, the complex impedance is denoted by a cap: $\hat{Z}$

Complex numbers



Representation of a complex number in a Gauss plain

Complex numbers are an extremely useful mathematical construct. Let's imagine a quadratic equation:

$$x^2 + x + 1 = 0$$

According to the relations for calculating the roots of a quadratic equation, we find that its roots are defined by the relation:

$$x_{1,2} = \frac{-1}{2} \pm \frac{\sqrt{-3}}{2}$$

In high school, it is usually taught that the square root of a negative number cannot be calculated, that is, that such an equation has no solution. The expression can be modified to contain only the square root of minus one:

$$x_{1,2} = \frac{-1}{2} \pm \sqrt{-1} \cdot \frac{\sqrt{3}}{2}$$

Similarly, the relation can be modified to solve any quadratic equation that has a negative discriminant. *The symbol i* is introduced for the complex unit (in electrical engineering and in signal theory, the symbol *j* is usually used to distinguish it consistently from electric current) defined by:

$$i := \sqrt{-1}$$

The complex unit has one very interesting feature:

$$i^2 = \left(\sqrt{-1}\right)^2 = -1$$

The solution to the equation can then be written in the form:

$$x_1 = \frac{-1}{2} + i \cdot \frac{\sqrt{3}}{2}$$
$$x_2 = \frac{-1}{2} - i \cdot \frac{\sqrt{3}}{2}$$

The numbers *x 1* and *x 2* are complex numbers. That part of them that is not a multiple of the complex unit is called the **real part of the complex number** , that part of them that is a multiple of the complex unit is called the **imaginary part of the complex number** .

Complex numbers can be viewed in another way, they are the sum of two real numbers, one of which is multiplied by a complex unit. The sum can be considered formal because a purely real and purely imaginary number cannot be added directly. This brings us to a more abstract view of complex numbers as a pair of real numbers with somehow defined addition, multiplication and division operations. In general, such a complex number *z* is written in **algebraic form** as:

$$z = a + i \cdot b$$

, where *a* and *b* are real numbers.

When we already have a pair of numbers, we can think of it as a point in a plane. Such a plane is actually used, it is called the Gaussian plane of complex numbers. We can then look at each complex number as a vector in a plane starting at the origin, whose first coordinate is the real part and the second coordinate is the imaginary part. However, the vector in the plane can also be expressed using the absolute value and the angle *φ* , which forms with the positive direction of the horizontal axis. Two equivalent notations are used for such an expression.

The **trigonometric form** of the complex number *z* :

$$z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi)$$

and the **exponential form** of the complex number *z* :

$$z = |z| \cdot e^{i \cdot \varphi}$$

The values are easily converted to each other, especially for the number *z=a+ib* :

$$|z| = \sqrt{a^2 + b^2}$$

Simple rules apply to calculating with complex numbers:

- $(a+ib)+(c+id)=(a+c)+i(b+d)$
- $(a+ib)\cdot(c+id)=ac+ibc+iad+i^2bd=(ac-bd)+i(bc+ad)$

Phasors

If it passes through the inductor or harmonic electric current through the capacitor, there is a phase shift between voltage and current, i.e. voltage and current reach their maximum at different times. The phase shift can only be 0-360°, so it is directly offered to be used to describe this process of complex numbers. We will choose one of the voltage or current values as the default, i.e. its phase will be zero, for the other we will indicate the phase shift including the sign. Complex written values are usually referred to as phasors and are hyphenated in the notation: $\hat{U}$, $\hat{I}$ and $\hat{Z}$. Even in phasors (in phasor space), Ohm's law applies, but care must be taken that calculations are performed with complex numbers:

$$\hat{Z}=\frac{\hat{U}}{\hat{I}}$$

Complex resistance

There is no phase shift between voltage and current on the resistance, so the phase shift of the complex resistance will be zero:

$$\hat{R}=R$$

Complex capacitance

On complex capacitance, the voltage lags behind the current by 90°, so the complex capacitance will have a phase of -90°. In component form, this means that the complex capacitance will be purely imaginary with a negative sign:

$$\hat{X}_C=-i\cdot\frac{1}{\omega C}$$

Complex inductance

On a complex inductance, the voltage leads the current by 90°, so the complex inductance will have a phase of 90°. In component form, this means that the complex capacitance will be purely imaginary with a positive sign:

$$\hat{X}_L=i\cdot\omega L$$

The complex impedance of a series connection of a resistor and a capacitor

Connecting impedances in series means that the impedances add up, so the form is simple:

$$\hat{Z}=\hat{R}+\hat{X}_C=R-i\cdot\frac{1}{\omega C}$$

Complex impedance of parallel connection of resistor and capacitor

Parallel connection of impedances leads to a more complex shape:

$$\hat{Z}=\frac{\hat{R}\cdot\hat{X}_C}{\hat{R}+\hat{X}_C}$$

After fitting and adjustments, the complex impedance has the following form:

$$\hat{Z}=\frac{R}{\omega^2C^2R^2+1}-i\cdot\frac{\omega CR^2}{\omega^2C^2R^2+1}$$

Graphical representation of impedance

Phasor diagram

A phasor diagram is a graphical representation of impedance at a single frequency. It is nothing more than plotting the complex impedance as a vector in a plane, which has the real part of the impedance as the x-coordinate and the imaginary part of the impedance as the y-coordinate. The phasor diagram makes it possible to easily carry out, for example, a graphical summation of impedances or to evaluate phase ratios very clearly.

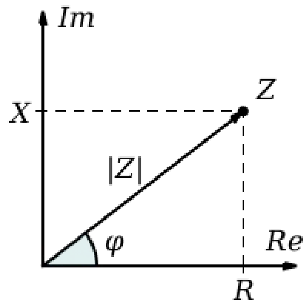
Impedance spectrum

When measuring impedance, it is quite often possible to know the dependence of impedance on frequency, because its shape can hide the information you are looking for, e.g. tissue hydration or the proportion of fat.

Mathematically, the impedance spectrum is actually a complex function:

$$\hat{Z} = \hat{Z}(\omega)$$

A certain technical problem is the graphical representation, since the point of such a graph is a pair of real and complex numbers. There are several ways to deal with this.



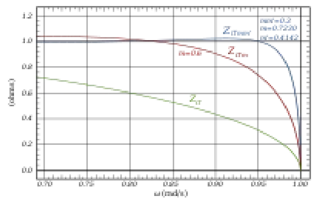
Impedance phasor diagram with real and positive imaginary component.

Real and imaginary spectrum

Impedance can be written as the sum of two real numbers - resistance *R* and reactance *X* , but in the case of frequency dependence, proceed analogously for the impedance spectrum and write impedance as the sum of two real functions:

$$\hat{Z}(\omega) = R(\omega) + i \cdot X(\omega)$$

Since both *R(ω)* and *X(ω)* are real functions, their graph can be easily plotted. The spectrum *R(ω)* is called the real impedance spectrum, the spectrum *X(ω)* is called the imaginary impedance spectrum. These spectra are used relatively little in practice, because visually less information is apparent from them than from other modifications.


A sample of three amplitude impedance spectra.

Amplitude and phase spectrum

he amplitude and phase spectrum is based on the exponential or trigonometric form of a complex number:

$$\hat{Z}(\omega) = Z(\omega) \cdot e^{-i\varphi(\omega)}$$

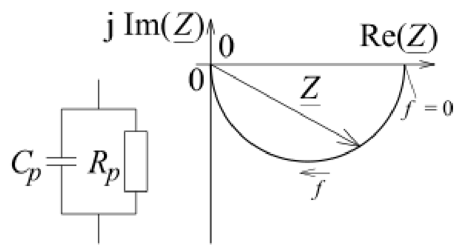
The real function *Z(ω)* is called the amplitude impedance spectrum, the real function *φ(ω)* is called the phase impedance spectrum. The amplitude spectrum can be easily interpreted as the total impedance at specific frequencies, while the phase spectrum adds information about the phase shift between voltage and current.

Sometimes a power spectrum is used, the name of which is derived from the processing of voltage signals. The frequency dependence of the square of the amplitude is displayed in the power spectrum.

Frequency response

The frequency characteristic is a two-dimensional graph in which the real component of the impedance is plotted on the horizontal axis and the imaginary component of the impedance on the vertical axis. In the case of general system theory, the plot is called a Nyquist plot. In the case of bioimpedance analysis, the negative imaginary component is usually plotted and is referred to as a Cole-Cole diagram.

The Cole-Cole diagram is a representation of the frequency dependence of the complex permittivity on frequency. It is named after the biophysicist brothers KS and RH Cole, who used it in the 1930s to study the complex impedance of tissues.



Frequency characteristic (Nyquist graph) of parallel connection of resistor and capacitor, i.e. the simplest model of the passive electrical properties of the organism.

Links

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- SENTA, Kubatova. *Biofot* [online]. [cit. 2011-01-31]. <https://uloz.to/!CM6zAi6z/biofot-doc>.