Biosignals from the point of view of biophysics - frequency spectrum of the signal

Frequency spectrum of the signal

In addition to the time course of the signal, a highly important characteristic is its frequency spectrum, i.e. the content of frequencies contained in it. For that reason, we have to clarify the basic **principle of the conversion between the time and frequency domains** , i.e. **the Fourier transform** , at the very beginning .

We see that the waveforms of the signal (18) or (19) contain a single frequency

f [Hz] = ω [rad/s] / 2 π [rad](20)

which follows directly from (13).

Stacking (superposition) of harmonic signals

Let's try what signal is created, for example, by combining two harmonic signals:

x(t) = u(t) + v(t)(21a)

where

u(t) = a. cos(α) = a. cos (ω. t)(21b)

v(t) = b. sin(α) = b. sin (ω. t)(22c)

We can imagine the signals u(t) and v(t) as the projections of two mutually perpendicular vectors of length *a* and *b* , rotating at a common angular velocity ω. Their **vector sum** will have a length given by the Pythagorean theorem

√(a2 + b2)(22a)

and phase φ, given by the ratio

tg (φ) = a / b(22b)

and will rotate with the same angular velocity . An important result that we would reach by generalizing this example is that the **sum of an arbitrary number of harmonic signals** with arbitrary amplitudes and phases, but **with the same frequencies** , is again a **harmonic signal of the same frequency** , whose amplitude and phase are given by the vector sum of the rotating vectors of the individual harmonic signals .

Another interesting experiment is the composition of two harmonic signals of **different frequencies** , of which the second frequency is an **integral multiple** of the first, **fundamental frequency** . The amplitudes and phases of both signals are again arbitrary. Let's choose as an example:

x(t) = x1(t) + x2(t)(23a)

kde x1(t) = a1. sin(α + φ1) = a1. sin(ω. t + φ1)(23b)

x2(t) = a2. sin(2α + φ2) = a2. sin(2ω. t + φ2)(23c)

The result will be a signal again with a period according to (17), but its **course will differ from** a sinusoidal, **harmonic** one . By experimenting with different amplitudes and phases, we can achieve different signal waveforms.

By using a larger number of harmonic signals with frequencies of one-, two-, three-, four-, etc. up to n-times of the fundamental frequency and their appropriate composition, we can very accurately approach a virtually arbitrarily chosen periodic signal course, and it is this procedure that is the basis of the harmonic or **Fourier synthesis** or Fourier development.

We call the sum a trigonometric Fourier series

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} [a_k \cos(k\omega t) + b_k \sin(k\omega t)] \quad (24)$$

It can be proved that using such a sum we can express practically any function, e.g. rectangular, triangular and any other.

A completely appropriate question is how to find the relevant weighting coefficients a k and b k so that we can use the series (23) to compose the necessary function.

Fourier (harmonic) analysis

The answer to the posed question is the opposite procedure to Fourier synthesis, and that is Fourier analysis, i.e. a procedure by which, on the contrary, any signal course can be **decomposed** into the above-mentioned sum (24). The corresponding coefficients can be found by integrating the product of the given function x(t) with the corresponding trigonometric function on the signal period interval:

$$a_k = \frac{2}{T_0}\int_0^{T_0} x(t).\cos(k\omega t)\mathrm{dt} \quad (25a)$$

$$b_k = \frac{2}{T_0}\int_0^{T_0} x(t).\sin(k\omega t)\mathrm{dt} \quad (25b)$$

Other examples of elementary waveforms:

- square wave signal
- unit impulse (Dirac distribution)
- unit jump (Heaviside function)
- triangular signal

Power spectrum of the signal (power spectrum)

Another important characteristic of a signal is its power spectrum, which answers the question to what extent the power of its individual components is represented in the frequency spectrum.

Electrical power is calculated as the **product of current and voltage** . **We can therefore calculate the instantaneous power of the signal** at each instant of time t as the **product of the instantaneous voltage and current** (instantaneous values of power, current and voltage are sometimes denoted by lowercase letters in contrast to the average values):

p(t) = u(t). i (t)(26)

The given relationship is true, but not very handy, because to calculate one time-varying quantity we need to calculate (and in practice therefore also measure) the product of two signals - one formed by current and the other by voltage. Usually, however, u(t) and ai(t) are not mutually independent quantities, but it is a signal that is consumed on some resistance or **load** , represented by a **real impedance** , **which we consider constant** within the given limits . **Ohm's law** is so universally valid that we can also use it for **variable quantities** :

u(t) = R. i(t)(27)

i(t) = u(t) / R(28)

Applying Ohm's law to the instantaneous power expression, we get the instantaneous power **expression** :

p(t) = u^2(t) / R = i^2(t). R(29)

We can use the mentioned relations for direct current and alternating current, here their special importance is shown in the case of calculating the instantaneous power of a signal of any waveform.**Assuming a constant load** , the consumed **power is proportional to the square of the voltage or current** . We consider this **quadratic dependence** of power on voltage or current even in cases where power consumption on the load is not explicitly mentioned.

Analogous relationships can also be derived in the case of **an acoustic signal** , when instead of electric voltage and current we consider the **instantaneous speed** of oscillating particles of the environment [m/s] and the **instantaneous pressure** [N/m 2]. Their product represents the **sound intensity** [W/m 2], therefore it has the character **of power** . In acoustics, we also consider **acoustic impedance** as one of the important characteristics of the environment, which – again within the given limits – can usually be **considered a constant** . Therefore, it is not surprising that even in the case of an acoustic signal, we find that the **instantaneous power is proportional to the square of the acoustic pressure oracoustic velocities** (not to be confused with the speed of sound propagation, here it is the instantaneous speed of oscillating particles of the environment).

We would reach similar results when investigating the course of other physical quantities that may carry some kind of signal. In this way, we arrive at the knowledge of the cardinal significance, namely that regardless of the specific physical representation, the **instantaneous power of the signal is proportional to the square of its instantaneous deviation** . In the case of **periodic signals** , we can deduce that their **power is proportional to the square of the signal amplitude .**

So I arrived at the way in which we can **calculate the power spectrum of any signal**: by harmonic analysis, so that instead of the waveform of the signal, **we calculate the waveform of its square.**

The Fourier transform has the important mathematical property that the individual frequency components are mutually **orthogonal** or independent. This has an enormously important consequence for practice, because even the powers of the individual components are independent of each other, and **the total power of the signal can be calculated as the sum of the powers of all its frequency components .** And further: if we have already calculated its frequency spectrum for a given signal course, in order to determine the power spectrum, it is not necessary to carry out the Fourier transformation once again on the square of its course, as we stated above, but it is enough to calculate the square of the amplitudes of the individual components of its frequency spectrum .

Links

Source

- HEŘMAN, Petr. *Biosignály z pohledu biofyziky.* 1. edition. Praha : Petr Heřman – DÚLOS, 2006. 64 pp.

Recommended literature

- AMLER, Evžen, et al. *Praktické úlohy z biofyziky I.* 1. edition. Praha : Praha: Ústav biofyziky 2. lékařské fakulty UK, 2006. ISBN 978-80-7305-113-6.
- HRAZDIRA, Ivo. *Biofyzika : učebnice pro lékařské fakulty.* 2. edition. Praha : Avicenum, 1990. ISBN 80-201-0046-6.
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- ROSINA, Jozef, et al. *Lékařská biofyzika.* 1. edition. Praha. 2000. 0 pp. ISBN 80-902318-5-3.
- NAVRÁTIL, Leoš – ROSINA, Jozef. *Biofyzika v medicíně.* 1. edition. Praha : Manus, 2003. 398 pp. ISBN 8086571033.
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