Biomechanics of fluids

Classification of liquids and their basic properties

Real and ideal liquids

First, it would be useful to define the important properties that distinguish liquids from other substances. Liquids have relatively good interparticle mobility, which, together with gases, which are a little better at this, makes them liquid. They do not have a fixed shape, and therefore conform to the shape of the container in which they have been placed. But at the same time, they no longer change their volume in a given container, whereas gases tend to expand and thus increase their volume. The relatively small distances between the molecules of liquids make them almost incompressible.

When we speak of an **ideal fluid**, we mean an ideal model of a real fluid. It helps us to understand the basic mechanisms of the phenomena occurring in liquids. An ideal fluid has zero viscosity (or is without internal friction), is therefore perfectly fluid and is also absolutely incompressible. Due to its low viscosity, the ideal liquid is close to e.g. water or even better liquid nitrogen at 77 K. Viscosity is the key property that makes the difference between a real and an ideal fluid, and it only becomes apparent in dynamics, not in statics. That is why it is the subject of several more paragraphs in the subsection on real fluids, or in the article on viscosity.

Ideal fluid

Ideal fluid flow

By flow we mean the movement of a fluid predominantly in one direction. The imaginary lines whose tangents show us the direction of the instantaneous velocity vector of the particle under consideration are called **streamlines**. In liquids, unlike solids, the relative positions of the particles change as they move. In principle, we distinguish between two types of flow:

1. **Stationary** (steady) - the velocity at a given point does not change with time
2. **Non-stationary** (unsteady) - the velocity at a given point varies with time

Steady flow of ideal fluid

From the previous definitions of the ideal fluid, we can conclude that the velocity of all particles at all points is the same across the cross-section of the tube, since the ideal fluid has zero viscosity. Furthermore, we can say that particles cannot accumulate anywhere in an ideal liquid because an ideal liquid is also perfectly incompressible. Stationary flow is characterized by the fact that it does not change with time, the velocity of all points in the cross-section of the tube will always be the same regardless of time. It follows, therefore, that in steady flow the same volume of liquid will flow through a given cross-section of the flow tube in the same time, i.e. the ideal liquid has a constant volume flow through the cross-section. The mathematical expression of this is the continuity equation.

Continuity equations

The volumetric flow through the surface S is defined in general as follows:

$$Q_V = \frac{\partial V}{\partial t}$$

where V is the volume of fluid flowing in a given time t. From the steady state assumption, the derivatives can be replaced by differences, i.e. we can go from infinitesimal changes to finite changes. The volumetric flow rate will then be the ratio of the volume of liquid that flowed through a given area in a given time:

$$Q_V = \frac{V}{t}$$

In the case of the ideal fluid, further simplifications can be made. Since the velocity of the flow is constant at all points, the volume of the liquid flowing in time t over the surface S will be given by the product of the surface and the distance d over which the liquid flowing in time t was located at time 0 at the point of the surface S. Since this is a motion with constant velocity v, we can write:

$$Q_V = \frac{V}{t} = \frac{S \cdot d}{t} = \frac{S \cdot v \cdot t}{t} = S \cdot v$$

In the case of a closed tube, the volume flow through **the entire** tube will be constant even with different cross-sections. It must therefore be true that:

$$S_1 \cdot v_1 = S_2 \cdot v_2$$

Thus, if the cross-section of the tube increases, the velocity of the liquid decreases and vice versa. The following figure illustrates the situation:

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Bernoulli's equation

Bernoulli's equation is a special case of the law of conservation of energy. This is because if we increase the velocity of the liquid by narrowing the tube, we increase the kinetic energy of the liquid. The law of conservation of energy implies that the potential energy must decrease. Relationship:

$$T + U = konst.$$

where T includes all forms of kinetic energy and U includes all forms of potential energy, is universally valid. The macroscopic manifestation of the kinetic energy of a flowing fluid is its velocity and hence hydrodynamic pressure. The potential energy of a fluid can be decomposed into two components, firstly the component corresponding to the potential energy in the gravitational field Eg and secondly the intrinsic potential energy of the interactions between the fluid particles E_p. It can thus be written:

$$E_k + E_g + E_p = konst.$$

The individual components can be expressed for a certain relatively small volume of liquid V of density ρ:

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\rho V v^2$$
$$E_g = mgh = \rho V h g$$
$$E_p = pV$$

The first two relationships are the physics curriculum for high school. The validity of the relationship for E^p can be shown by a simple consideration of how the energy of a fluid enclosed in a cylinder with a sliding piston changes if we start to apply a force to the piston. The equations can be combined, the constant volume can be converted to the right-hand side to obtain the familiar form of Bernoulli's equation:

$$\frac{1}{2}\rho v^2 + h\rho g + p = konst.$$

The immediate consequence is that if the speed increases due to the narrowing of the tube, the pressure decreases. Bernoulli's equation cannot be applied to the flow of blood through blood vessels in the human body because it is too closely tied to the ideal fluid in its derivation. However, human blood is a real fluid, it is even a non-Newtonian fluid (viscosity depends on flow velocity). Another deviation is the elasticity of blood vessels, where e.g. large arteries in systole use part of the kinetic energy of the blood to increase their own potential energy (they stretch) and in diastole give most of this energy back as kinetic energy of the blood.

Real fluid

While the ideal liquid could be imagined as a number of perfectly slippery billiard balls completely filling the container, in the case of a real liquid this idea is no longer sufficient. The particles of a real liquid exert attractive and repulsive forces on each other, which ultimately leads to non-negligible internal friction and to the distance between the particles not being "the smallest possible". The macroscopic manifestations are then viscosity and compressibility.

Viscosity

Viscosity is a dynamic property of real fluids. The dynamic property here means that it only manifests itself when the fluid is flowing. It is actually an expression of the degree of internal friction in a liquid. Viscosity, more precisely **dynamic viscosity**, is defined as the constant of proportionality η (Pa.s) in Newton's law of viscosity:

$$\sigma = \eta \frac{dv}{dx}$$

Newton's law in a nutshell says that if two streamlines (i.e. surfaces of the same velocity) flow in a liquid close to each other, more precisely infinitely close at a distance dx, the tension between these layers will be proportional to the ratio of the difference between their velocities (which will also be infinitesimal dv) and the distance, i.e. actually the derivative of the velocity according to the distance of the streamlines. The dynamic viscosity is a constant of proportionality. Since viscosity is a measure of internal friction, the higher the numerical value of dynamic viscosity, the more resistance the fluid will put up to the flow. The viscosity decreases with increasing temperature.

Sometimes relative viscosity is used, i.e. the ratio of the viscosity of the liquid under study to that of the reference liquid. For example, if water is used as the reference fluid, the relative viscosity of plasma will be around 2 and that of blood around 4.

Since viscosity is defined as a constant of proportionality of Newton's law of flow, fluids for which the dynamic viscosity is constant are referred to as Newtonian fluids. Liquids for which the viscosity is dependent on the flow velocity are called non-Newtonian. These are usually macromolecular suspensions, examples being chocolate, blood, milk or dissolved starch.

Flow of real liquids

Real fluid can flow in two ways:

- **laminárně**
- **turbulentně**

In laminar flow, the different layers of the fluid do not mix and flow side by side. The layers move in sequence and form a characteristic profile as they flow through the tube, with the particles in the middle layer having the highest velocity, which gradually decreases towards the walls and is minimal at the walls. The fluid flows laminarly at lower velocities, at higher velocities the fluid begins to form eddies and the flow becomes turbulent. In turbulent flow, the streamlines curve chaotically and the fluid layers mix. The transition from laminar to turbulent flow is quite complex. The Reynolds number is used to estimate whether, under given conditions, the flow of a Newtonian fluid will be laminar or turbulent. If the Reynolds number is less than one critical value, the flow is laminar, if it is greater than the other critical value, the flow is turbulent. For a flow in a tube of circular cross-section, the Reynolds number is defined:

$$Re = \frac{\rho D v}{\eta}$$

where Re is the Reynolds number, ρ is the density of the liquid, D is the diameter of the tube (vessel), v is the velocity of the liquid flow and η is the dynamic viscosity of the substance, all in basic units. If the value is greater than 1000, the flow will be turbulent.

The following two figures illustrate the difference between laminar and turbulent flow:

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Clinical significance of turbulence:

- Turbulence often occurs in anemia due to decreased blood viscosity.
- Noises may be generated in turbulent flows. Turbulent flow can also be encountered in practice, e.g. when measuring blood pressure. The inflated cuff narrows at the point of constriction, the radius of the vessel and the blood velocity increases. This leads to turbulence and the so-called Korotkoff phenomena, which can be heard in the stethoscope.
- Turbulence also stresses the vessel wall. It is believed that turbulent flow may chronically traumatize the endothelium and thus be one of the factors involved in the development of atherosclerosis.

Laminar flow in blood vessels

The velocity profile of blood in blood vessels varies according to the caliber of the vessel. In arterioles, the velocity profile has the character of an elongated rotational paraboloid, i.e. the velocity is maximum in the axis of the arteriole and decreases sharply to very small values towards the edges of the arteriole. With increasing vascular translucency, the velocity profile becomes more and more flattened.

Hagen - Poiseuille's Law

When the flow of a real fluid is steady, the flow can be described by the Hagen-Poissel law, which takes into account the geometric proportions of the tube and the viscosity of the fluid. The volumetric flow is directly proportional to the fourth power of the tube radius and the pressure change and inversely proportional to the viscosity and the length of the tube:

$$Q = \frac{\pi r^4 \Delta p}{8 L \eta}$$

where r is the radius of the tube, η is the dynamic viscosity, Δp is the pressure difference at the beginning and end of the tube and L is the length of the tube.

For the application to the bloodstream it is necessary to note the following fact: Since the law applies to steady laminar flow in a rigid tube, this formula cannot be used quantitatively. However, it can be used to estimate dependence and proportionality. A slight modification of the Hagen-Poiseullo law, using the analogy of volumetric flow with Ohm's law, gives us a formula for the peripheral resistance imposed by blood vessels. It still shows that the resistance of a vessel decreases with the fourth power of its radius. The more we narrow the radius of the vessel, the more we increase the peripheral resistance. The peripheral resistance is inversely proportional to the volume flow rate, therefore increasing the resistance will decrease the volume flow rate.

If we consider the fluid flow as analogous to an electric current and the pressure difference as analogous to a voltage drop, we can write "Ohm's law" in the form:

$$Q = \frac{\Delta p}{R}$$

where resistance (sometimes hydraulic resistance, peripheral resistance) R *(Pa.m⁻³)* is defined as:

$$R = \frac{8\eta L}{\pi r^4}$$

Diffusion

Diffusion is characterised as the spontaneous passage of particles from a location of higher concentration to a location of lower concentration. This happens until a state of dynamic equilibrium is reached in the system. In this state, although the diffused particles pass in both directions, their concentration does not increase in the long term on either side. In addition to the concentration gradient, the electrical gradient also plays an important role. Therefore, in other words, it can be said to be an equilibration of electrochemical potentials. The diffusion process results in a uniform dispersion of all components of the system in the case where diffusion is not affected in the system under study.

Diffusion occurs in all three states and its rate depends on the kinetic energy of the diffusing particles. Therefore, it is fastest in the gaseous state with a high kinetic energy and slowest in the solid state with a low kinetic energy. The rate of diffusion is described by Fick's laws (see below) and its measure is the diffusion coefficient. Diffusion is an irreversible process associated with an increase in entropy, which means that the most disordered dispersion of diffusing particles is achieved. At absolute zero, diffusion does not take place because the motion of the elementary particles ceases.

Factors affecting diffusion

State

As mentioned above, diffusion occurs easily in gases and liquids because both types of states have a high kinetic energy. In addition, the diffusion process can be accelerated by mechanical action, for example by mixing.

In solids, diffusion is very difficult and time consuming. However, it is also the only way in which a substance can be transported, which is why diffusion is most important in this state.

Temperature

At absolute zero (T=0K) the motion of elementary particles stops, therefore diffusion does not work. Its effect then increases with increasing temperature. This factor is described by the Brownian equation of motion:

$$\frac{2KTt}{\xi t} = \frac{kTt}{3\pi\mu r}$$

Material type

Diffusion is impeded by the crystal lattice material. The tighter the lattice arrangement, the more difficult diffusion is. The bonding between the atoms also makes diffusion more difficult. The thicker it is, the more difficult diffusion is.

Diffusion flow

To quantify diffusion, the diffusion flux **J** is used, which gives the molar amount *n* of the substance under study that passes through the surface *S* in time *t*. Importantly, the surface *S* must be perpendicular to the direction of diffusion. In the differential form, i.e. with an infinitesimal change in time and a corresponding infinitesimal amount of substance flowing through, the definition has the form:

$$J = \frac{1}{S}\frac{dn}{dt}$$

Fick's Laws

Fick's laws describe the rate of diffusion, not diffusion per se. These are 2 laws expressed by 2 equations.

Fick's First Law

Fick's first law describes the diffusion flux at steady state, i.e. assuming that the concentration gradient $\partial c/\partial x$ does not change over time. The movement of molecules is in the direction of decreasing concentration. Mathematically, the diffusion flux is directly proportional to the concentration gradient with a constant of proportionality D *(m².s⁻¹)*

(diffusion coefficient). In the one-dimensional case, e.g. diffusion through a thin capillary or diffusion through a membrane, the concentration gradient is equal to the derivative of the concentration with respect to the corresponding spatial coordinate:

$$J = -D\frac{\partial c}{\partial x}$$

The partial derivative symbol here indicates that the concentration is variable not only in space but also in time.

The diffusion coefficient depends on the temperature T, the dynamic viscosity of the liquid η and the size of the diffusing particles R according to the following relation:

$$D = \frac{kT}{6\pi\eta R}$$

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Fick's Second Law

As mentioned above, diffusion is characterized by a change in gradient. The concentration of the diffused substance therefore depends not only on the spatial coordinate (see Fick's first law) but also on time. This is why the second Fick's law must be added to the first Fick's law.

The second Fick's law characterizes the changes in concentration over time. We introduce it for non-stationary diffusion. When describing diffusion in one spatial dimension (linear diffusion), the second Fick's law takes the form:

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Osmosis

Osmosis is a method of passive transport in which the system tries to reach thermodynamic equilibrium. It is a specific type of diffusion across a semipermeable membrane that readily permeates the solvent, but practically does not permeate the solute at all. Thus, equilibration of chemical potentials occurs by the passage of solvent from a solution with a lower solute concentration (hypotonic solution) into a solution with a higher solute concentration (hypertonic solution).

If the membrane is unyielding, the result is an increase in compartmental pressure. The magnitude of osmosis is determined by the difference in osmotic pressures on either side of the semipermeable membrane, with the magnitude of osmotic pressure determined by the van't Hoff equation, which gives the dependence of osmotic pressure on concentration and temperature. When the temperature of the starting materials is increased by 10 °C the value of the rate constant is increased to twice, which is based on the Arrhenius equation. It is for very dilute solutions (i.e. deviations from van´t Hoff's law increase with increasing molecular weight of the solute). A solution of molar concentration c has osmotic pressure Π at temperature T:

$$\Pi = cRT$$

The osmotic pressure of intracellular fluid and blood plasma is about 770 kPa. If the solution has a lower concentration, it is a hypotonic solution. The cell, when immersed in a hypotonic solution, takes up water and enlarges (called cell swelling). Plasmoptysis can occur in an animal cell. In this case, the plant cell is protected by a solid cell wall and therefore does not burst, but the pressure of the protoplast on the cell wall increases (called turgor). The animal cell maintains a low internal solute concentration by actively pumping out ions; the protozoa, for a change, periodically ejects water that accumulates in the cell (pulsating vacuole). The plant cell is protected against cell swelling by a rigid cell wall.

Osmotic phenomena

Plasmolysis

It is caused by a hypertonic environment. It means shrinkage of the cell due to lack of solvent. The plant cell is subject to it, and since it has a solid cell wall, the cell membrane detaches from the wall and the protoplast content is reduced.

Plasmorhiza

It is caused by the hypertonic environment around the animal cell. Since the animal cell does not have a solid cell wall, shrinkage occurs due to loss of solvent inside the cell.

Plasmoptysis

It is caused by a hypotonic environment. It means the rupture of the cell due to excess solvent. The animal cell is subject to it, the plant cell is immune to plasmoptosis.

Links

Related articles

- 1. Fick's Law
- Biomechanics of blood circulation
- Bernoulli's equation
- Hagen-Poiseuille law
- Osmotic pressure, osmosis
- Osmosis
- Viscosity

External links

- Patobiomechanika a Patokineziologie Kompendium (http://biomech.ftvs.cuni.cz/pbpk/kompendium/biomechanika/index.php)
- Transport látek - vysvětlení obecného principu difuze (https://is.muni.cz/do/rect/el/estud/prif/js11/fyz_chem/web/dynamika/transport.htm)
- Mechanika kapalin a plynů (http://fyzika.jreichl.com/main.article/view/109-mechanika-kapalin-a-plynu)
- Fickův zákon (http://old.vscht.cz/fch/prikladnik/prikladnik/p.9.1.html)
- w:cs:Difuze Difuze
- Odvození rovnice Brownova pohybu (http://christie21.blog.cz/1212/brownuv-pohyb-rovnice)

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